A General Constructive Approach to Matrix Converter Stabilization

Yao Sun, Mei Su, Xing Li, Hui Wang, and Weihua Gui

***Abstract*—The stability of a matrix converter is investigated in both motor mode and generator mode. Inspired by the impedance ratio criterion, a general constructive method is proposed to stabilize matrix converter. Based on the analysis, it is possible to change the admittance of matrix converter by modifying the output reference voltage and rectifier modulation vector according to the constructive method. As a result, the instability which is caused by negative impedance could be eliminated. Meanwhile, the stability regions are enlarged. Furthermore, the underlying principles of the constructive method are stated in detail and several instructive examples, which comply with these principles, are demonstrated. The essence of some other existing methods used to improve the stability of matrix converter is unveiled, and the difference and similarity between the existing methods and the proposed ones are also discussed. The correctness and feasibility of the proposed method are verified by simulations, as well as experiments.**

***Index Terms*—Constructive method, matrix converter, stability, stabilization.**

I. Introduction

THE MATRIX converter has attracted more and more attention recently due to many advantageous features such as bidirectional power flow, controllable input power factor, sinusoidal input and output currents, and compactness in structure [1]. In the past decade, many research works have been conducted on the modulation strategies [2]–[4], reliability issues [5], [6], stability problem [7]–[10], and new topologies derived from the basic matrix converter, such as indirect matrix converter [11]–[13]. Among them, the stability problem caused by the input *LC* filter is of relatively less concern in matrix converter, but it is very essential for its normal operation, especially in the weak power grid.

The reasons for installing an input filter before the matrix converter are twofold: on one hand, it mitigates the input current harmonics; on the other hand, it is used to assist the commutation of switching devices to assure the normal operation of the converter. Generally, a second-order *LC* filter is used. Since the parasite resistance of the *LC* filter is low, the resonant mode of the input filter is easily excited by the harmonics from power grid and the matrix converter itself.

Especially, in the case of heavy constant power loads, the matrix converter may behave as a negative impedance. If the inherent damping ratio of the *LC* input filter is too small, the negative impedance of the matrix converter will lead to instability. Usually, this problem can be solved in two different ways, i.e., passive and active ones.

The passive way [14] is to increase the damping coefficient of the system by designing a damped input filter, for example, putting a proper damping resistor in parallel with the filter inductor. This method is reasonable when the system undergoes high voltage and low current. However, in low-voltage and high-current applications, it is desirable to install an *RC* damping branch, instead of the parallel resistor, in parallel with the filter capacitor, to provide sufficient damping without consuming much power.

The active way is to use additional algorithms to increase the damping coefficient. In that case, the instability phenomenon could be avoided without adding extra components and consuming excessive power. To improve the stability of matrix converter, the method proposed in [8] used a digital low-pass filter in synchronous reference frame to smoothen the input voltages. In [7], the small-signal stability analysis is introduced to study the stability of matrix converter, considering the effects of the sample-and-hold circuit and control delay. In [9], the large-signal stability of matrix converter is studied further. In [15], a method, filtering the amplitude as well as the phase angle of input voltages, is proposed to further improve the stability of matrix converter, and experiments with some different loads are carried out. The approach of utilizing a PLL has proved to be feasible, when it was used to enhance the system stability in [16]. Essentially, it is a variant of the input voltage phase angle filtering method. The stability of matrix converter when driving a doubly fed induction generator is investigated in [17]. A technique by modifying the input reference currents to increase the damping and improve the system stability is presented in [18]–[19]. It assumes that the input current of matrix converter is controllable completely. Thus, the modulation strategy is more difficult to implement, and most of the existing modulation algorithms are not feasible for it.

According to the impedance ratio criterion [20], [21], if the output impedance of the input filter is much lower than the input impedance of the power converter, the effect of the filter on the system can be ignored. Then, if an algorithm can be applied to modify the input impedance of matrix converter, the instability caused by the input filter could be avoided. Therefore, a simple and general method to improve system stability based on a

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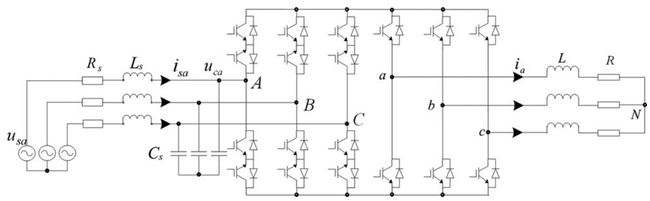


Fig. 1. Schematic diagram of an indirect matrix converter.

constructive method is proposed in this paper. The procedures and principles of the method are also stated in detail. In addition, the essence of the existing methods is explored, and some comparison analyses are carried out between the proposed method and the existing ones.

This paper is organized as follows. In Section II, the mathematical modeling of matrix converter is introduced. The stability of matrix converter is studied under open-loop and closed-loop modulation strategies. In Section III, the reasons of matrix converter's instability are analyzed; furthermore, a general constructive technique is used to improve the stability under motor operation as well as generator operation. In Section IV, simulations and experiments are carried out to verify the effectiveness of the technique proposed. In Section V, the conclusions of this paper are drawn.

II. Matrix Converter Modeling and Modulation

A. Modeling

With regard to indirect matrix converter and traditional matrix converter (matrix converter), there are no major differences between their mathematical models. Thus, without loss of generality, the indirect matrix converter shown in Fig. 1 is used for study in this paper. The whole system consists of a power supply, an input filter, a matrix converter circuit, and a set of *RL* load. All the parameters of power line are lumped into R_s and L_s. To investigate its stability, at first, the state-space averaging method is applied to model the system. For convenience, the dynamic equations in the stationary reference frame, in terms of complex vectors, are given as follows:

$$L_s \frac{d\vec{i}_s}{dt} = \vec{u}_s - \vec{u}_c - R_s \vec{i}_s \tag{1}$$

$$C_s \frac{d\vec{u}_c}{dt} = \vec{i}_s - \vec{i}_r \tag{2}$$

$$L \frac{d\vec{i}}{dt} = \vec{u} - R\vec{i} \tag{3}$$

$$\vec{u} = 1.5(\vec{u}_c \bullet \vec{m}_r)\vec{m}_i \tag{4}$$

$$\vec{i}_r = 1.5(\vec{m}_i \bullet \vec{i})\vec{m}_r \tag{5}$$

where $\vec{m}_r$ represents the unit rectifier modulation vector rotating at the angular frequency ω_s of power supply, $\vec{m}_i$ is the inverter modulation vector rotating at ω_o, and $|\vec{m}_i| \leq \sqrt{3}/3$; $\vec{u}_s$, $\vec{u}$, and $\vec{i}_r$ are the grid voltage, the output voltage, and input current of the matrix converter, respectively. Equations (1) and (2), and (3) represent the dynamics of the input filter and the *RL* passive load, respectively. The coupling relations between the rectifier and inverter of matrix converter are described by (4) and (5), reflecting the distinctive feature of matrix converter.

B. Open-Loop Modulation

Assume that the impedance of the power line is so small that the voltage drops across it could be neglected. If voltage sensors are installed before the input filter, the measured voltage could be regarded as the grid voltage $\vec{u}_s$ approximately. To realize the unity input power factor, the rectifier modulation vector is set to $\vec{m}_r = \vec{u}_s/|\vec{u}_s|$. For a given output reference voltage $\vec{u}^*$, considering the fact $\vec{u}_s \approx \vec{u}_c$(it is reasonable when the input filter is well designed), the inverter modulation vector $\vec{m}_i$ could be solved according to (4) by replacing $\vec{u}_c$ with $\vec{u}_s$. As the obtained $\vec{m}_i$ does not depend on the information of $\vec{u}_c$, it is called the open-loop modulation.

Based on the orientation principle, (1) and (2) at the input side of the matrix converter are written in a reference frame rotating at a supply angular frequency ω_s, with *D*-axis along the direction of vector $\vec{u}_s$. And the load dynamic (3) is written in a reference frame rotating at output voltage angular frequency ω_o with *D*-axis along the direction of vector $\vec{u}$. Then, decompose them into *D*-axis and *Q*-axis as follows:

$$L_s \frac{di_{sd}}{dt} = |\vec{u}_s| - u_{cd} + L_s\omega_s i_{sq} - R_s i_{sd} \tag{6}$$

$$L_s \frac{di_{sq}}{dt} = -u_{cq} - L_s\omega_s i_{sd} - R_s i_{sq} \tag{7}$$

$$C_s \frac{du_{cd}}{dt} = i_{sd} - i_{rd} + C_s\omega_s u_{cq} \tag{8}$$

$$C_s \frac{du_{cq}}{dt} = i_{sq} - i_{rq} - C_s\omega_s u_{cd} \tag{9}$$

$$L \frac{di_d}{dt} = u_d - Ri_d + L\omega_o i_q \tag{10}$$

$$L \frac{di_q}{dt} = u_q - Ri_q - L\omega_o i_d. \tag{11}$$

According to the open-loop modulation and the unity input power factor requirement, we could rewrite the input current and output voltage of the matrix converter as follows: $i_{rd} = ri_d$, $i_{rq} = 0$, $u_d = ru_{cd}$, and $u_q = 0$, where $r = |\vec{u}^*|/|\vec{u}_s|$. After substituting them into (8)–(11), we can get the following system equation:

$$\Lambda \dot{x} = Ax + D \tag{12}$$

where

$$\Lambda = \mathrm{diag}\{\, L_s \quad L_s \quad C_s \quad C_s \quad L \quad L \,\}$$
$$x = [\, i_{sd} \quad i_{sq} \quad u_{cd} \quad u_{cq} \quad i_d \quad i_q \,]^T$$
$$D = [\, |\vec{u}_s| \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T$$

$$A = \begin{bmatrix} -R_s & L_s\omega_s & -1 & 0 & 0 & 0 \\ -L_s\omega_s & -R_s & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & C_s\omega_s & -r & 0 \\ 0 & 1 & -C_s\omega_s & 0 & 0 & 0 \\ 0 & 0 & r & 0 & -R & L\omega_o \\ 0 & 0 & 0 & 0 & -L\omega_o & -R \end{bmatrix}.$$

As can be seen that the system matrix A is a constant matrix and could be decomposed into the sum of a negative semidefinite matrix and a skew-symmetric matrix, it is not difficult to find that the system is stable according to [22] when the output reference voltage is constant (i.e., r is a constant). Despite this, along with the increase of r and output power, a pair of closed-loop dominant poles move toward the imaginary axis of the complex plane; consequently, the resonances due to the weakly damped modes may still occur.

The open-loop modulation has the advantage of easy implementation. However, if the impedance of power line and filtering inductor is large enough, and especially in the case of heavy load, there may be a certain deviation between the actual input voltage of matrix converter and grid voltage inevitably. As a result, a large deviation may exist between the actual output voltage and the desired value, which will affect the output performance seriously. In addition, the stability analysis aforesaid is no longer valid because the measured voltage could no longer be regarded as the grid voltage. For these reasons, if the power grid is stiff enough and the input filter is designed appropriately, and meanwhile, there exists a closed-loop current control on the load side, the open-loop modulation is still a desirable approach.

C. Closed-Loop Modulation

The purpose of modulation is to make the actual output voltages of the matrix converter as close to the desired ones as possible. If the voltage sensors are mounted before the input filter, which means that the measured voltages are different from the ones fed into the matrix converter, the errors will be reflected on the output side of the matrix converter directly. The best way to overcome the errors is to install the sensors behind the input filter to sample the actual input voltages $\vec{u}_c$. According to (4) and (5), for the given reference, $\vec{m}_i$ and $\vec{m}_r$ are solved on line, and then the output voltage could be compensated simultaneously in accordance with the varying input voltages. In this paper, we denote such a way as the closed-loop modulation.

To assure unity input power factor, the rectifier modulation vector is defined as $\vec{m}_r = \vec{u}_c/|\vec{u}_c|$, and the desired output voltage is set to $\vec{u}^* = (u_d^* + ju_q^*)\cdot e^{j\omega_o t}$. According to (4) and (5), the input currents of the matrix converter in the synchronous rotating reference frame are given by

$$i_{rd} = \frac{u_{cd}}{u_{cd}^2 + u_{cq}^2}(u_d^* i_d + u_q^* i_q) \tag{13}$$

$$i_{rq} = \frac{u_{cq}}{u_{cd}^2 + u_{cq}^2}(u_d^* i_d + u_q^* i_q). \tag{14}$$

Then, substituting (13) and (14) into (8) and (9) leads to the following state equation:

$$\Lambda\dot{x} = A_c x + D_c \tag{15}$$

where Λ and $x$are the same as those in (12), but

$$A_c = \begin{bmatrix} -R_s & L_s\omega_s & -1 & 0 & 0 & 0 \\ -L_s\omega_s & -R_s & 0 & -1 & 0 & 0 \\ 1 & 0 & -\dfrac{u_d^* i_d + u_q^* i_q}{u_{cd}^2 + u_{cq}^2} & C_s\omega_s & 0 & 0 \\ 0 & 1 & -C_s\omega_s & -\dfrac{u_d^* i_d + u_q^* i_q}{u_{cd}^2 + u_{cq}^2} & 0 & 0 \\ 0 & 0 & 0 & 0 & -R & L\omega_o \\ 0 & 0 & 0 & 0 & -L\omega_o & -R \end{bmatrix}$$

$$D_c = \begin{bmatrix} u_{sd} & u_{sq} & 0 & 0 & u_d^* & u_q^* \end{bmatrix}^T.$$

It could be found that the system matrix A_c includes state variables. For such a nonlinear system, it is difficult to determine the stability of the system according to its structure. Therefore, the small-signal analysis method is used here. Before linearization of (15) around the steady-state operating point, the steady-state operating point should be solved first. Note that, the variable, with a bar over it in this paper, represents its corresponding steady ones at the steady-state operating point. To simplify the calculation, let

$$u_{cq} = 0; \quad u_q^* = 0. \tag{16}$$

The linearization of (15) leads to the following small-signal equations:

$$\Delta\dot{x} = \Lambda^{-1}\tilde{A}_c\Delta x \tag{17}$$

where

$$\tilde{A}_c = \begin{bmatrix} -R_s & L_s\omega_s & -1 & 0 & 0 & 0 \\ -L_s\omega_s & -R_s & 0 & -1 & 0 & 0 \\ 1 & 0 & \dfrac{u_d^*\bar{i}_d}{\bar{u}_{cd}^2} & C_s\omega_s & -\dfrac{u_d^*}{\bar{u}_{cd}} & 0 \\ 0 & 1 & -C_s\omega_s & -\dfrac{u_d^*\bar{i}_d}{\bar{u}_{cd}^2} & 0 & 0 \\ 0 & 0 & 0 & 0 & -R & L\omega_o \\ 0 & 0 & 0 & 0 & -L\omega_o & -R \end{bmatrix}.$$

From the last two lines of $\tilde{A}_c$, it can be found that the load part is a stable isolated system. Therefore, to investigate the stability of the system (17), we only need to consider the fourth-order submatrix which does not include the load dynamics. According to the structure characteristic of the submatrix, it is not difficult to find the positive element $u_d^*\bar{i}_d/\bar{u}_{cd}^2$ on the diagonal is the main reason that poses a threat to the system stability.

D. Origin of Instability

For simplicity, a dc/dc converter with an input filter is taken as an example to introduce the instability problem caused by negative impedance. The converter could be regarded as an input conductance, and its equivalent circuit is shown in Fig. 2.

The diagram in Fig. 2(a) can be transformed to the equivalent circuit diagram as shown in Fig. 2(b). The input current can be derived as follows:

$$i = u_s\frac{Y}{L_s C_s s^2 + R_s C_s s + 1}\frac{1}{1 + z_o Y} \tag{18}$$

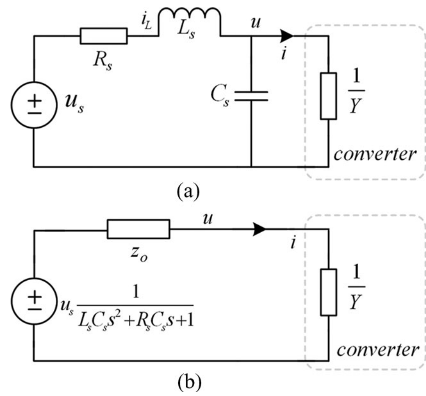


Fig. 2. Circuit diagram of a dc/dc converter: (a) circuit diagram and (b) equivalent circuit diagram.

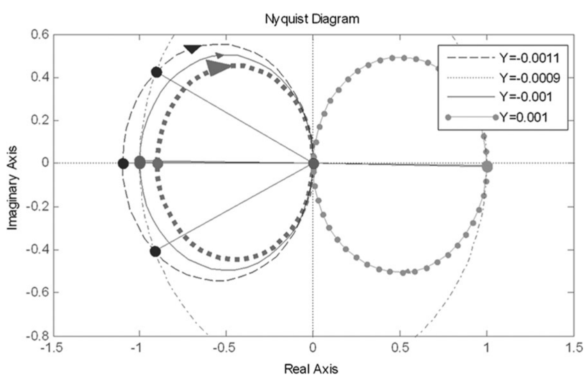


Fig. 3. Nyqusit plots.

where Y is the input admittance and z_o is the output impedance

$$z_o = \frac{R_s + L_s s}{L_s C_s s^2 + R_s C_s s + 1} \tag{19}$$

If Y is stable, the system will be stable under the condition that the nyquist contour of $z_o Y$ does not encircle the point $(-1, 0)$. If Y is a real number and $Y > 0$, the system will be always stable. If $Y < 0$, the necessary and sufficient condition to guarantee the stability is

$$|Y| < Y_c = \frac{1}{\max\{R_s, L_s/(R_s C_s)\}}. \tag{20}$$

Otherwise, the system may lose its stability. From the nyquist plots in Fig. 3, we could verify all the proposals aforesaid, where $Y_c = -0.001$; the parameters used are $R_s = 0.1\ \Omega$, $L_s = 3\,\text{mH}$, $C_s = 30\ \mu\text{F}$.

Assume $u \cdot i = p$; if p is a positive constant, then it is the constant power load [23].

Then, its input admittance is

$$Y = -\frac{p}{\bar{u}^2}. \tag{21}$$

In this case, if p is too large so that condition (20) cannot be met, the converter will lose its stability.

E. Constructive Method in Motor Mode

To investigate the stability of matrix converter, the converter could also be regarded as an admittance. As the standard matrix converter is a three-phase system, the input admittance could be written on *D*-axis and *Q*-axis by the coordinate transformation and voltage orientation. Under the closed-loop modulation, according to (13) and (14), and (16), taking the first-order linear approximation of the Taylor series, the mall-signal linear equivalent equations could be derived as

$$\Delta i_{rd} = \frac{u_d^*}{\bar{u}_{cd}} \Delta i_d - \frac{\bar{u}_d^* \bar{i}_d}{\bar{u}_{cd}^2} \Delta u_{cd}$$

$$\Delta i_{rq} = \frac{\bar{u}_d^* \bar{i}_d}{\bar{u}_{cd}^2} \Delta u_{cq}.$$

Because $\bar{u}_d^*$ is a constant, $\Delta i_d = 0$. After taking Laplace transformation, the input admittances at the operating point are described as follows:

$$\frac{\Delta i_{rd}(s)}{\Delta u_{cd}(s)} = -\frac{u_d^* \bar{i}_d}{\bar{u}_{cd}^2} \tag{22}$$

$$\frac{\Delta i_{rq}(s)}{\Delta u_{cq}(s)} = \frac{u_d^* \bar{i}_d}{\bar{u}_{cd}^2}. \tag{23}$$

It is observed that the real parts of admittances on *D*-axis and on *Q*-axis are complementary. Whatever operation mode is, there always exists a negative admittance.

As reported in the previous section, the negative impedance (or conductance) may lead to instability. However, if the input admittances can be modified to satisfy the stability conditions by using additional algorithms, the stabilization of matrix converter may be achieved.

In this section, a constructive method, which is used to modify the input conductance, will be explained. Its basic idea is to construct a proper correction term for the control input of matrix converter based on the stability requirement.

For example, if the output reference voltage on *D*-axis is modified as

$$u_d^{**} = u_d^* + f(u_{cd}) \tag{24}$$

where u_d^{**} is the revised output reference voltage, $f(u_{cd})$ is the correction term. An analogous process to the derivation of (22) and (23) is made; then, the modified input admittances on *D*-axis and *Q*-axis could be derived as follows:

$$\frac{\Delta i_{rd}(s)}{\Delta u_{cd}(s)} = \frac{u_d^* \bar{i}_d - \bar{u}_{cd} \bar{i}_d \partial f_{u_{cd}} + \bar{i}_d \bar{f}}{-\bar{u}_{cd}^2} + H(s) \tag{25}$$

$$\frac{\Delta i_{rq}(s)}{\Delta u_{cq}(s)} = \frac{(u_d^* + \bar{f})\bar{i}_d}{\bar{u}_{cd}^2} \tag{26}$$

where $\partial f_{u_{cd}} = \partial f(u_{cd})/\partial u_{cd}$, $H(s) = (u_d^* + \bar{f})L(s)\partial f_{u_{cd}}/\bar{u}_{cd}$. $H(s)$ results from the loading effect. Usually, input admittance of power converter is related to load, control method, converter itself, and so on. $L(s)$ is a transfer function which depends on the specific loads. When an *RL* load is considered, $L(s)$ is shown as follows:

$$L(s) = (Ls + R)/[(Ls + R)^2 + L^2\omega_o^2]. \tag{27}$$

Next, the key question becomes how to construct the correction term. The basic requirements of the construction method are 1) there is no deviation between the actual output voltages and their references in steady state; 2) the deviation must be small enough during transient state; 3) the stability must be improved.

According to the aforementioned requirements, three principles to construct the modification terms are given as follows:

1) $f(u_{cd})$ should be designed to guarantee that the system is stable;
2) $f(u_{cd})$ should be small, and $\bar{f}(u_{cd}) = 0$;
3) $f(u_{cd})$ does not have to be an algebraic equation; instead, it could be a dynamic system such as a filter.

According to the first principle (it could also be called as stability principle), if $f(u_{cd})$ is an algebraic equation, $\partial f_{u_{cd}}$ should be greater than zero and large enough to make the system stable in motor mode. For the second principle, $f(u_{cd})$ should be as small as possible not to affect output performance.

According to the previous principles, several intuitively constructive examples are given as follows:

Case 1:

$$f(u_{cd}) = k \cdot (u_{cd} - \bar{u}_{cd}) \tag{28}$$

where $\bar{u}_{cd}$ is the steady state of u_{cd}. It could be obtained by solving the steady algebraic equation when all the parameters are available. In engineering, $\bar{u}_{cd}$ could also be approximated by the rated grid voltage in the case of no large voltage disturbances, because the voltage drop across the input filter is very small.

From (28), we get $\partial f_{u_{cd}} = k$ and $\bar{f}(u_{cd}) = 0$. Thus, this method complies with all the principles mentioned previously when k is selected properly. If the condition $u_d^* \bar{i}_d - \bar{u}_{cd} \bar{i}_d k < 0$ holds by choosing a proper k, then the stability is guaranteed. However, if k is too large, the modification term will cause a large deviation from its reference which may lead to modulation index saturation and affect the output performance of the converter. So, the selection of k is a tradeoff between stability and dynamic performance.

Case 2:

$$\begin{aligned} f(u_{cd}) &= \ell^{-1}\left\{k[u_{cd}(s) - \frac{u_{cd}(s)}{\tau s + 1}]\right\} \\ &= k \cdot \ell^{-1}\left\{\frac{\tau s}{\tau s + 1} u_{cd}(s)\right\} \end{aligned} \tag{29}$$

where $\ell^{-1}\{\}$ represents the inverse Laplace transform. The idea behind this constructive method is that $\bar{u}_{cd}$ could be got through a low-pass filter. Such a method is preferred from practical point of view, because the actual value of $\bar{u}_{cd}$ is not available due to unknown and uncertain load. As for the stability principle, it will be investigated by root locus plots in the following. It is not difficult to find that $f(u_{cd})$in (29) complies with the second principle from the point of view of frequency response.

Case 3:

$$f(u_{cd}) = k \cdot \bar{u}_{cd}\left[\left(\frac{u_{cd}}{\bar{u}_{cd}}\right)^{\gamma} - 1\right] \tag{30}$$

where $\gamma > 1$, when $u_{cd} < \bar{u}_{cd}$; $\gamma < 1$ when $u_{cd} > \bar{u}_{cd}$; $\gamma = 1$, while $u_{cd} = \bar{u}_{cd}$. This is a nonlinear constructive method, in which the correction term $f(u_{cd})$ will be smaller than that in Case 1 during the transient state, while the same k is used in both cases. According to (30), in steady state, $\bar{f}(u_{cd}) = 0$, which means that the second principle holds. In this case, $\partial f_{u_{cd}} = k$. k could be selected the same as that in Case 1 to comply with the stability principle.

For the closed-loop modulation, assume that the constructive method (28) is used. Considering the modified output reference voltage in (13) and (14), the linearization of the system (6)–(11) around the operating point leads to the following small-signal system equation:

$$\Delta\dot{x} = \Lambda^{-1}\tilde{A}_{c1}\Delta x$$

where

$$\tilde{A}_{c1} = \begin{bmatrix} -R_s & L_s\omega_s & -1 & 0 & 0 & 0 \\ -L_s\omega_s & -R_s & 0 & -1 & 0 & 0 \\ 1 & 0 & \dfrac{u_d^*\bar{i}_d - \bar{u}_{cd}\bar{i}_d \cdot k}{\bar{u}_{cd}^2} & C_s\omega_s & -\dfrac{u_d^*}{\bar{u}_{cd}} & 0 \\ 0 & 1 & -C_s\omega_s & -\dfrac{u_d^*\bar{i}_d}{\bar{u}_{cd}^2} & 0 & 0 \\ 0 & 0 & k & 0 & -R & L\omega_i \\ 0 & 0 & 0 & 0 & -L\omega_i & -R \end{bmatrix}.$$

Observing the system matrix $\tilde{A}_{c1}$, if k is big enough such that $u_d^*\bar{i}_d - \bar{u}_{cd}\bar{i}_d \cdot k < 0$, and if both the fifth and sixth rows of matrix $\tilde{A}_{c1}$ and Λ are multiplied by $u_d^*/(\bar{u}_{cd}k)$, we will get a new system matrix $\tilde{A}'_{c1}$ and a new diagonal matrix Λ'. Then the system equation could be rewritten as $\Delta\dot{x} = (\Lambda')^{-1}\tilde{A}'_{c1}\Delta x$. Since $\tilde{A}'_{c1}$ could also be decomposed into a negative definite and a skew-symmetric matrix, according to [22], it could be proved that the matrix converter system is stable with the proposed constructive method.

When the constructive method (29) is used, since its system matrix has lost the special structure characteristic as $\tilde{A}_c$, it is not easy to derive a simple stability condition. To investigate the stability of method (29), the root locus technique is used. There are two parameters k and τ in method (29); therefore, two partial root locus diagrams are given. The closed-loop poles with an increasing k which is varied from 0 to 2 and a fixed $\tau = 0.1 \times 10^{-4}$ are shown in Fig. 4(a). It can be found that one pair of poles could be shifted from right half-plane to right half-plane by increasing k. The root locus with a fixed $k = 0.5$ and an increasing τ which is varied from 0.1×10^{-3} to 1×10^{-3} is shown in Fig. 4(b). As can been seen from Fig. 4(b) the system can be stabilized and a damping ratio is improved by increasing τ.

From the viewpoint of frequency response, the methods in (28) and (29) have the same frequency response only in higher frequency band. Therefore, the method in (29) only suppresses high-frequency resonance only, compared with method (28). The capability to improve the stability is degraded by the low-pass filter; however, it helps to avoid saturating the modulation index further [24]. The effect of k is the same as that in (28). For parameter τ, from Fig. 4(b), if τ is too small, the damping effect is small; if τ is too large, it will lead to a large modification

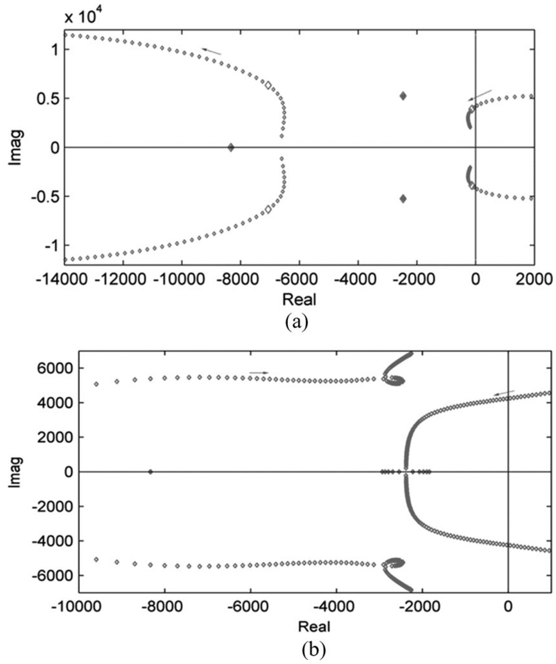


Fig. 4. (a) Closed-loop poles of the system as a function of k when τ is fixed. (b) Closed-loop poles of the system as a function of τ when k is fixed.

during transient. As a result, it degrades the dynamic performance of matrix converter. So, a tradeoff must be made between the stability and performance.

In this section, just the output reference voltage on *D*-axis is modified. In fact, the output reference voltage on *Q*-axis could also be modified to improve the stability, for example, $u_q^{**} = u_q^* + f(u_{cd})$, which is especially effective when the output current i_q is higher than i_d. The idea behind modifying u_q^* to improve stability is the same as that of modifying u_d^*, and the selection of correction term $f(u_{cd})$ is also the same as that described previously. Up to now, all the constructive methods discussed previously are only suitable for motor mode. However, when a matrix converter works in generator mode, the input admittance of the matrix converter on *Q*-axis behaves as negative impedance, and then the proposed methods will be invalid to stabilize the matrix converter.

F. Constructive Method in Generator Mode

It is well known that the matrix converter has two control inputs, i.e., the rectifier modulation vector and inverter modulation vector. It has been found that modifying the output reference voltage could change the input admittance on *D*-axis in previous sections. In this section, we will discuss how to modify the input admittance of matrix converter on *Q*-axis to improve the stability.

Assuming that the input voltage vector $\vec{u}_c$ could be written as $|\vec{u}_c|\, e^{j\theta}$, considering unity input power factor (neglecting the additional phase shift caused by the input filter), then the rectifier modulation vector angle is θ; based on the same idea as (24), the rectifier modulation vector angle is modified as follows:

$$\theta^* = \theta + g(\theta) \tag{31}$$

where θ^* is the revised rectifier modulation vector angle and $g(\theta)$ is the correction term.

According to (4) and (5), the input current vector is obtained as follows:

$$\vec{i}_r = \frac{u_d^* i_d + u_q^* i_q}{|\vec{u}_c|^2} \vec{u}_c e^{jg(\theta)}. \tag{32}$$

Decomposing $\vec{i}_r$ in the synchronous rotating reference frame rotating at ω_s, we have

$$i_{rd} = \frac{u_d^* i_d + u_q^* i_q}{u_{cd}^2 + u_{cq}^2}[u_{cd}\cos(g(\theta)) - u_{cq}\sin(g(\theta))] \tag{33}$$

$$i_{rq} = \frac{u_d^* i_d + u_q^* i_q}{u_{cd}^2 + u_{cq}^2}[u_{cq}\cos(g(\theta)) + u_{cd}\sin(g(\theta))] \tag{34}$$

where

$$\theta = \theta_s + \tan^{-1}\left(\frac{u_{cq}}{u_{cd}}\right)$$

and θ_s is the angle of the synchronous rotating reference frame.

In the steady state, assume that condition (16) holds, linearizing (33) and (34) around the operating point and taking Laplace transformation; then, the input admittances are obtained as follows:

$$\frac{\Delta i_{rd}(s)}{\Delta u_{cd}(s)} = -\frac{u_d^* \bar{i}_d}{\bar{u}_{cd}^2} \tag{35}$$

$$\frac{\Delta i_{rq}(s)}{\Delta u_{cq}(s)} = \frac{u_d^* \bar{i}_d[1 + \partial g(\theta)/\partial\theta]}{\bar{u}_{cd}^2}. \tag{36}$$

From the aforesaid equations, the input admittance on *D*-axis is greater than zero in generator mode. However, the input admittance on *Q*-axis is less than zero, which may results in the instability of matrix converter. From (36), it is found that the input admittance on *Q*-axis could be changed by constructing a proper $g(\theta)$. The principles to construct the correction term are the same as those stated in the previous section.

A simple constructive method is to let

$$g(\theta) = k_\theta \cdot (\theta - \bar{\theta}) \tag{37}$$

where $\bar{\theta} = \theta_s$. If $\bar{\theta}$ is available, this method is effective to improve the stability when k_θ is less than -1. The condition $k_\theta < -1$ indicates that the input admittance on *Q*-axis is larger than zero, which implies that the stability principle is complied with. In steady state, $\bar{\theta} = \theta$, i.e., $\bar{g}(\theta) = 0$, which complies with the second principle.

Another example of constructive method is

$$g(\theta) = k_\theta \cdot \ell^{-1}\left\{\frac{\tau_\theta s}{\tau_\theta s + 1}\theta(s)\right\}. \tag{38}$$

The main idea behind this method is similar to that in Case 2 mentioned previously. There exists a proper k_θ to meet the stability principle, which could be validated by the root locus method or other numerical analysis methods. According to the final value theorem, obviously, $\bar{g}(\theta) = 0$, which complies with the second principle.

G. Previous Works From the Constructive Approach Viewpoint

According to the method proposed in [8], its rectifier modulation vector $\vec{m}_r$ is set to be $\vec{u}_c/|\vec{u}_c|$. According to (4), if $\vec{u}_c$ is replaced by $\tilde{u}_c$, then the solved inverter modulation vector $\vec{m}_i$ is $2\vec{u}^*/3\,|\tilde{u}_c|$. Substituting the solved $\vec{m}_i$ and $\vec{m}_r$ into (5), the input currents of matrix converter could be calculated as follows:

$$i_{rd} = \frac{u_{cd}}{|\vec{u}_c|\,|\tilde{u}_c|}(u_d^* i_d + u_q^* i_q) \tag{39}$$

$$i_{rq} = \frac{u_{cq}}{|\vec{u}_c|\,|\tilde{u}_c|}(u_d^* i_d + u_q^* i_q) \tag{40}$$

where $\tilde{u}_c$ is the output of a digital low-pass filter in the synchronous rotating reference frame with the input $\vec{u}_c$. After some mathematical manipulations, the input admittance on *D*-axis is

$$Y_D(s) = -\frac{u_d^*}{\bar{u}_{cd}^2}\frac{1}{\tau s+1}\left[\bar{i}_d - u_d^* \cdot L(s)\cdot \tau \cdot s\right] \tag{41}$$

where $Y_D(s)$ denotes the input admittance on *D*-axis, and τ is the time constant of the digital low-pass filter. In fact, this method could be described as

$$u_d^{**} = u_d^* + k_1 \cdot \ell^{-1}\left\{\frac{\tau s}{\tau s+1}u_{cd}(s)\right\} \tag{42}$$

where $k_1 = u_d^*/\ell^{-1}\{\frac{1}{\tau s+1}u_{cd}(s)\}$.

The second term on the right side of (42) is nonlinear, and after linearization it becomes $(u_d^*/\bar{u}_{cd})\cdot \ell^{-1}\{(\tau s/\tau s+1)u_{cd}(s)\}$. Comparing (42) with the constructive method in (29), it is found that they are the same in structure but with different gains: the gain k_1 in (42) depends on the ratio between u_d^* and the input voltage, but the gain *k* in (29) could be selected freely according to the requirements. It is safe to say the constructive method is a generic method. In fact, if the damping effect of the method in (42) is not good enough, k_1 could be magnified properly or the correction term in (28) could be combined into (42) as follows:

$$u_d^{**} = u_d^* + k_1 \cdot \ell^{-1}\left\{\frac{\tau s}{\tau s+1}u_{cd}(s)\right\} + k\cdot(u_{cd} - \bar{u}_{cd}). \tag{43}$$

Literature [15] investigated the stability problem of matrix converter by filtering the phase angle of input voltage vector. It could be written as

$$\theta^* = \theta - \ell^{-1}\left\{\frac{\tau_\theta s}{\tau_\theta s+1}\theta(s)\right\}. \tag{44}$$

Compared with (38), the only difference between them lies in k_θ; the method proposed in [15] lets k_θ be constant -1 actually; however, the gain k_θ could be selected freely. The solution based on PLL in [16] is equivalent to filtering the phase angle of input voltage vector through a second-order low-pass filter.

H. Constructive Method in Both Modes

Based on the aforementioned analysis mentioned, assume that methods (28) and (37) are used; when the converter works in motor mode, we need that the conditions $k \geq u_d^*/\bar{u}_{cd}$ and $k_\theta \geq -1$ hold. Generally, only the input admittance on *D*-axis should be modified. When working in generator mode, we need the conditions $k \leq u_d^*/\bar{u}_{cd}$ and $k_\theta \leq -1$ hold. In this case, only the input admittance on *Q*-axis should be changed. If the direction of the reference power flow, which is not the real power flow, is used as a trigger signal to switch proper constructive method, the matrix converter will also work correctly in both modes. In such a way, the unnecessary switching between two the constructive methods could be avoided during transients due to the error to detect the sign of real power flow.

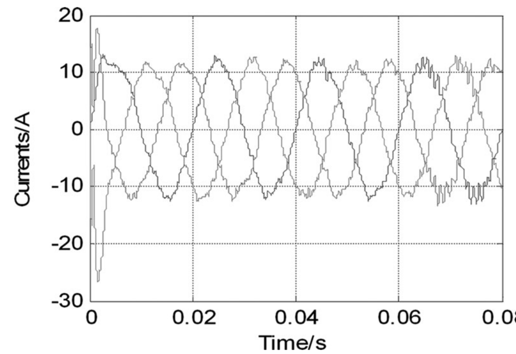


Fig. 5. Input currents of matrix converter.

III. Simulation and Experimental Results

A. Simulation

To verify the validity of the proposed scheme, two simulations are carried out in Matlab/Simulink. In the first one, a fixed output voltage control is considered; in the second one, the closed-loop output current control is investigated.

In the first simulation, the setup is as follows: the power supply is 220 V (RMS)/50 Hz, input filter $L_s = 3$ mH, $C_s = 10\ \mu$F, and $R_s = 0.01\ \Omega$, and its configuration is shown in Fig. 1. The passive *RL* load is as follows: $R = 1\ \Omega$ and $L = 0.6$ mH. The sampling frequency is set to 10 kHz.

The constructive method expressed in (28) is applied before 0.04 s, where $k = 0.5$; during the interval 0.04–0.06 s, the method in (42) is used, but k_1 is amplified two times; for the last 0.02 s, the method proposed in [8] is used. All the time constant is set as 0.8 ms in this simulation, and its reference output voltage is set to 60 V (peak)/50 Hz. The simulation results of the input currents are shown in Fig. 5. The total harmonic distortion (THD) with the three methods is 5.77%, 7.26%, and 9.89%, respectively. It is found that the method proposed in this paper is effective to damp the oscillations in the input currents. From the waveforms during the interval 0.06–0.08 s, it can be found that the oscillation with the method in [8] is more severe; the reason is that its equivalent gain is fixed and not enough. As a result, it could not provide enough damping in this situation for the system. The method in (42) is an equivalent variant of the method proposed in [8], but it is more flexible and its equivalent gain could be modified according to the specific requirements. For instance, in this experiment, when its equivalent gain is multiplied by 2, it could inject more damping into the system, and then the related THD of the input current is lower.

To verify the proposed method which is able to further improve the stability limits of matrix converter, the method in (43)

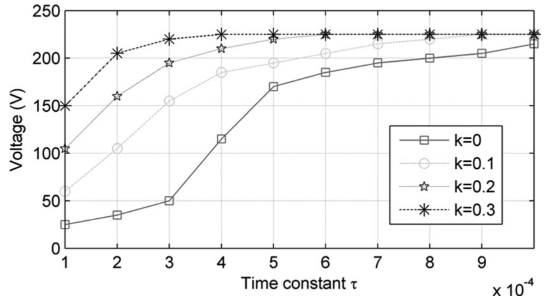


Fig. 6. Stability limits of matrix converter output voltage (peak) against time constant τ of a low-pass filter with different gain *k*.

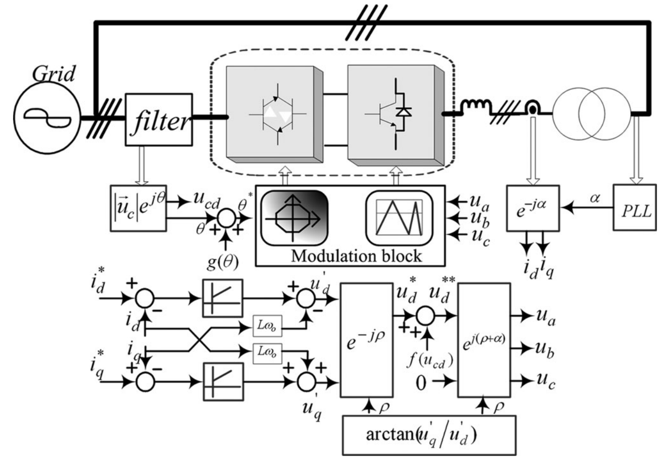


Fig. 7. Control block diagram for a matrix converter running both in motor mode and in generator mode.

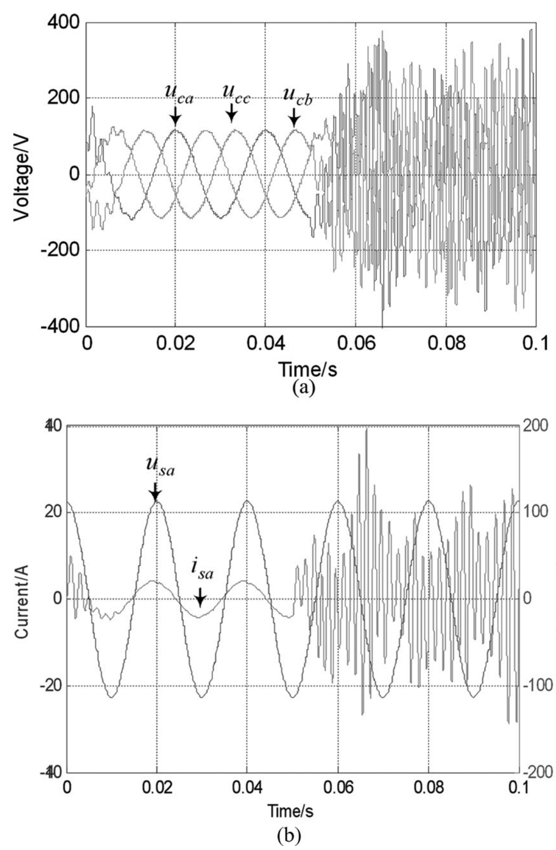


Fig. 8. Simulation results of a matrix converter system when only output reference voltages are modified in motor mode. (a) Input voltages. (b) Input current and grid voltage.

is used in the closed-loop modulation. A numerical analysis is carried out to plot the boundary of matrix converter instability under different parameters τ and *k*. The parameters used in the numerical analysis are the same as those in the first simulation; and its output voltage frequency is 25 Hz. As can be seen from Fig. 6, the area below the curve represents the stable operating area. When $k = 0$, the method coincides with method proposed in [8]; it can be found that the stability region becomes larger with increasing *k*.

In order to test the behavior of matrix converter in both motor mode and generator mode, a second simulation experiment is conducted. Its configuration is shown in Fig. 7. The whole system mainly includes an input filter, a main circuit of matrix converter, an output filter, and a step-up transformer. Two proportional integral (PI) controllers in the synchronous rotating reference frame are used to regulate output currents based on the voltage-oriented control. The matrix converter works as a grid-connected inverter. The injected active power and reactive power could be controlled by regulating the injected currents. The injected energy comes from the power grid and comes back to the power grid through the matrix converter and transformer. The operation mode can be selected by changing the sign of the reference current on *D*-axis (the direction of power flow). The classic current space-vector modulation is used to realize rectification and the carrier-based modulation is used for inverter. The detailed system modeling is given in the Appendix.

The parameters used in this simulation are as follows: input filter $L_s = 3$ mH, $C_s = 30$ μF, and $R_s = 0.1$ Ω. The filtering inductance at the output side is $L_g = 3$ mH; the turn ratio of transformer is 110/220, and the grid voltage is 80 V (RMS).

Before $t = 0.05$ s, the reference current i_d is set to 8 A (peak); matrix converter works in motor mode; after $t = 0.05$ s, i_d is changed to -8 A; then matrix converter works in generator mode.

Fig. 8(a) shows the input voltages waveforms of the matrix converter under the condition that only the method in (29) is used. It could be found that the matrix converter becomes unstable once turning into generator mode. The corresponding input currents are depicted in Fig. 8(b).

In Fig. 9(a) and (b), the input voltages and currents of the matrix converter are illustrated when only using the method in (38). It is found that the matrix converter cannot work in motor mode correctly, but behaves well in generator mode.

In motor mode, the method in (29) is applied. While in generator mode, the method in (38) is implemented. In this way, the simulation results are shown in Fig. 10. The output currents on *D*-axis and *Q*-axis are shown in Fig. 10(a); it can be seen that the references are tracked quickly. The input voltages and currents of the matrix converter are shown in Fig. 10(b) and (c), respectively. And the matrix converter works well in both modes.

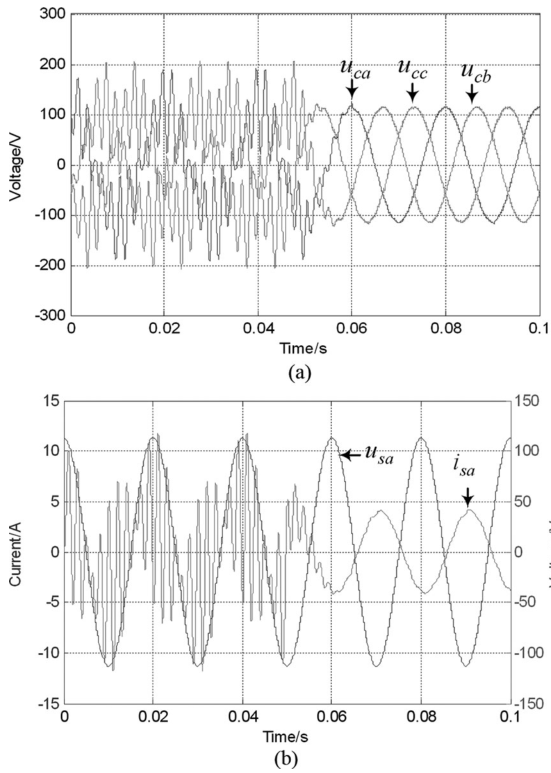


Fig. 9. Simulation results of a matrix converter system when only a rectifier modulation vector is modified in generator. (a) Input voltages. (b) Input current and grid voltage.

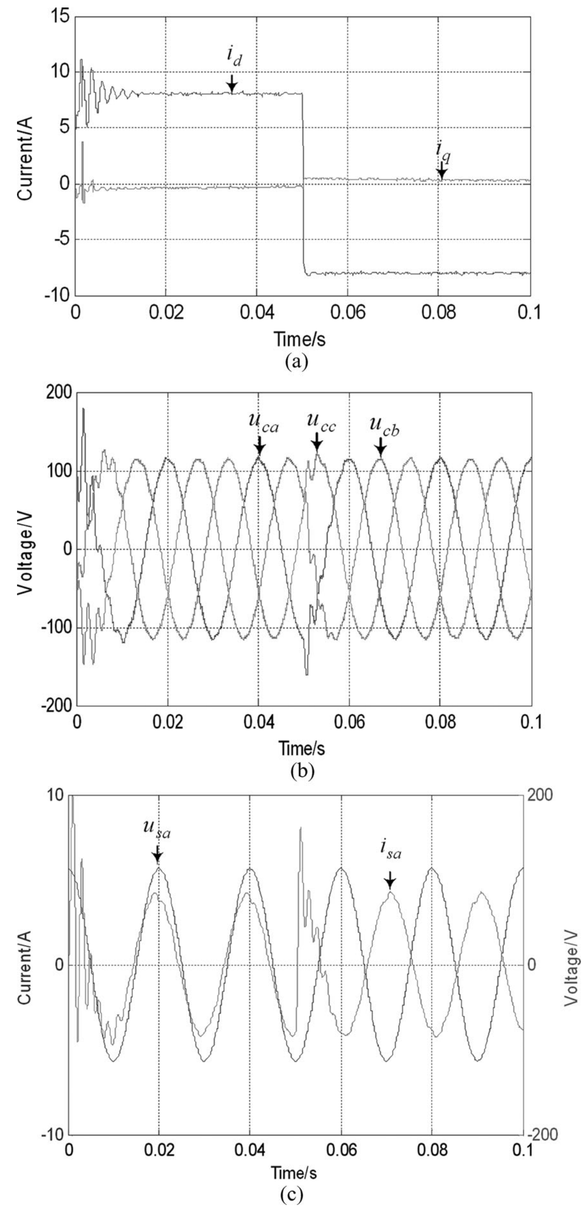


Fig. 10. Simulation results of a matrix converter system when the output reference voltage is modified only in motor and rectifier modulation vector is modified only in generating mode. (a) Output currents. (b) Input voltages. (c) Input current and grid voltage.

To analyze the behavior of matrix converter with both methods in (29) and (38) being used simultaneously in motor mode or generating mode, some experiments are carried out. In Fig. 11(a), during the time 0.04–0.05 s (motor mode), both methods are used simultaneously; as a result, oscillation in the current occurs in this interval. In Fig. 11(b), during the interval 0.05–0.06 s (generator mode), both methods are used simultaneously; oscillation in current can be found in this interval too. Based on the aforesaid results, it can be found that the contribution of both the methods for stabilizing matrix converter conflicts with each other, which is in good agreement with theoretical analysis.

To prove the effectiveness of method (29) under grid disturbance and load disturbance, some other simulations have been performed. During the time 0–0.05 s, the grid voltages are superposed by 5 V sine wave with 1000 Hz, and the output reference current i_d is set to be 8 A; during the interval 0.05–0.1 s, the disturbance in grid voltages is removed, and the output reference current i_d is changed to be 10 A. The correction item $f(u_{cd})$ is shown in Fig. 12(a). It can be found that the correction item is large during the transient, and converges to a small steady value quickly. It also can be found that the influence on the performance due to the high-frequency sine disturbance is small. Fig. 12(b) shows the waveform of the output active current. Fig. 12(c) shows the waveform of grid voltage with periodic disturbance.

B. Experiments

An indirect matrix converter prototype rated 10 kW has been developed in the laboratory to verify the proposed methods. Its controller board is mainly composed of a floating-point DSP (TMS320F28335) and a field-programmable gate array (EP2C8J144C8 N). Both the sampling and switching frequency is 10 kHz.

The same as the simulation part, two experiments are conducted to validate the effectiveness of the proposed methods. In the first experiment, three tests are carried out, showing the impact of the time constant of low-pass filter and extra degree of freedom k on the stability of matrix converter. The purpose of the second experiment is to prove that the proposed

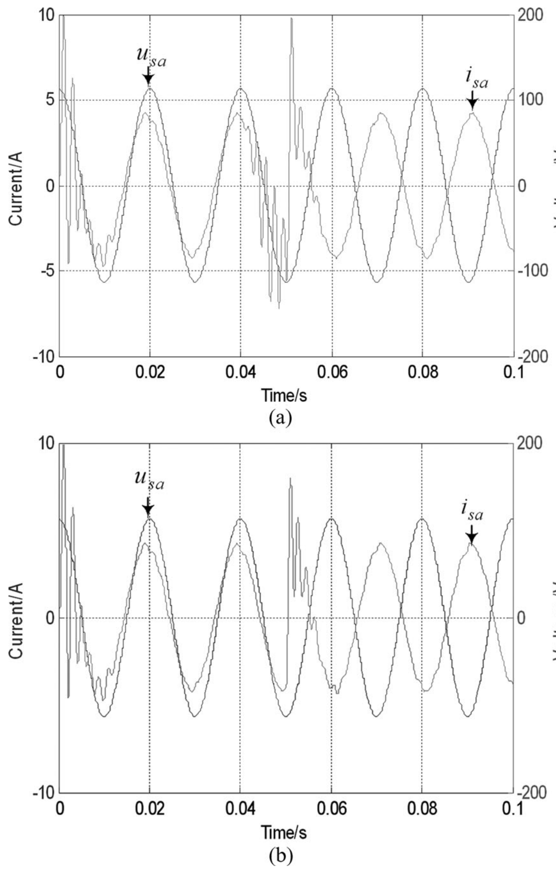


Fig. 11. Grid voltage and input current. (a) During interval 0.04–0.05 s (motor mode), both modifications are used. (b) During interval 0.05–0.06 s (generating mode), both modifications are used simultaneously.

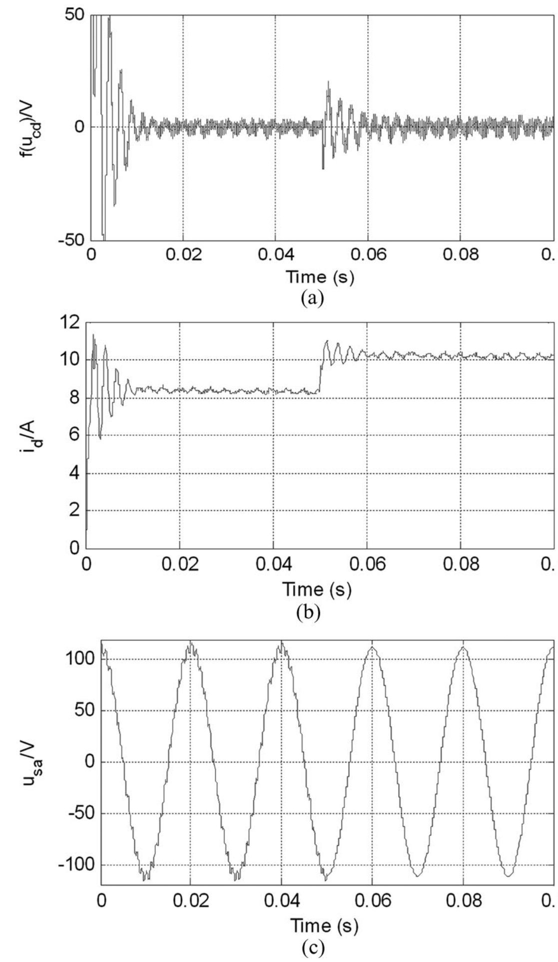


Fig. 12. Output voltage correction term with grid voltage distortion and varying output load currents. (a) Output voltage correction term $f(u_{cd})$. (b) Wave form of output active current. (c) Wave form of grid voltage u_{sa}.

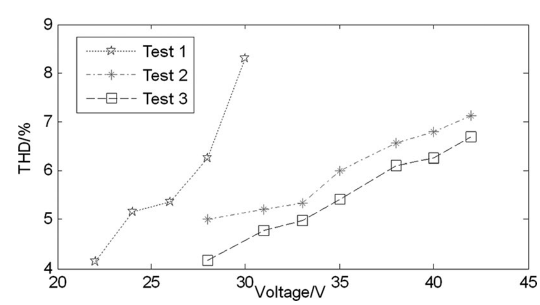


Fig. 13. THD of input voltages versus output reference voltages in three tests.

constructive methods are able to stabilize matrix converter operating in motor as well as generator mode.

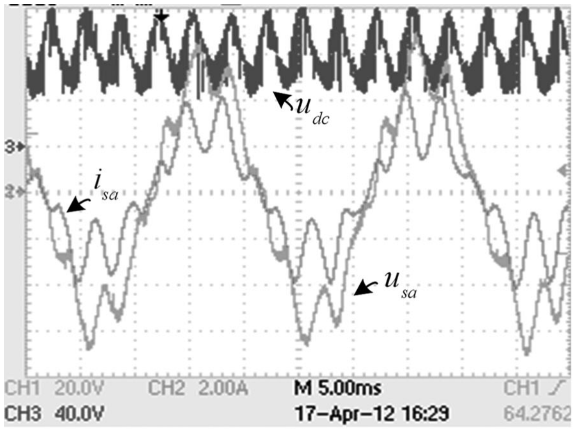
In the first experiment, the parameters used are as follows: the voltage of power supply 40 V (RMS)/50 Hz, input filter $L_s = 3$ mH, $C_s = 30\ \mu$F, *RL* load $R = 5.1\ \Omega$, $L = 0.6$ mH. Filter reactor L_s does not include the inductance in power supply.

Three tests are done in this experiment. Test 1 uses the control algorithm in (42) with $\tau = 0.1$ ms; the control algorithm in Test 2 is also based on (42), but with $\tau = 1$ ms; Test 3 applies the method in (29) with $k = 0.6$ and $\tau = 1$ ms.

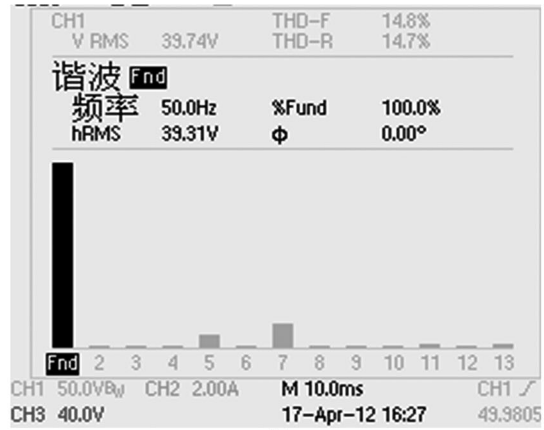
Because it is not easy to forecast the occurrence of instability accurately in practice, in the first experiment, the reference output voltages are increased step by step for safety considerations. The results obtained in the three tests are shown in Fig. 13, which are the THD of input voltage versus different reference output voltages. Clearly, the result of Test 3 is the best one. According to the comparisons between the results in Test 1 and Test 2, it can be seen that, by increasing the time constant of low-pass filter, the stability region could be extended greatly. In addition, due to the extra degree of freedom *k*, the power quality in Test 3 is better than that in Test 2.

With regard to Test 1, the result in Fig 13 shows that, when the output voltage reaches 33 V (peak), the THD of input voltages becomes large. At this operation point, some more detailed results are illustrated in Fig. 14(a) and (b). Fig. 14(a) indicates that the system is close to the instability point. Its related FFT spectrum analysis of the input voltage is shown in Fig. 14(b).

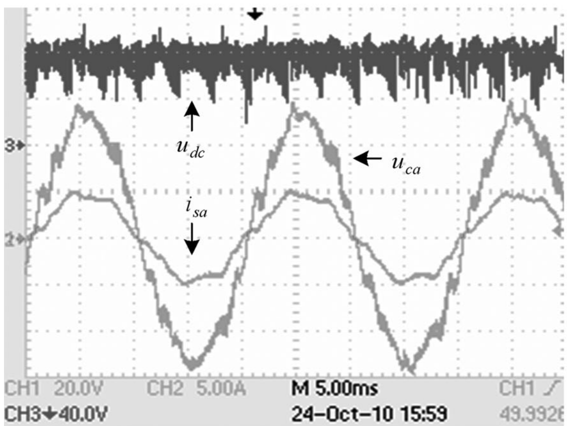
Some comparisons are made between Test 2 and Test 3 by setting the reference output voltage to 42 V in the two tests. The measured input voltage, current, and dc-link voltage of matrix converter are shown in Fig. 14(c) and (e), respectively. The associated FFT spectra of the input voltage are shown in Fig. 14(d) and (f), respectively, to verify the effect of damping of different stabilization methods. It can be seen that the result in Test 3 is better than that in Test 2.



(a)

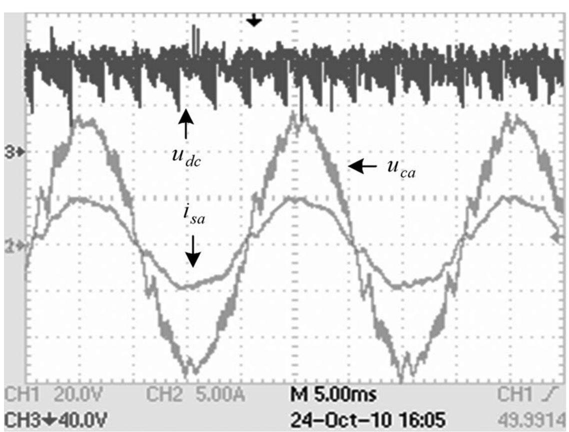


(b)

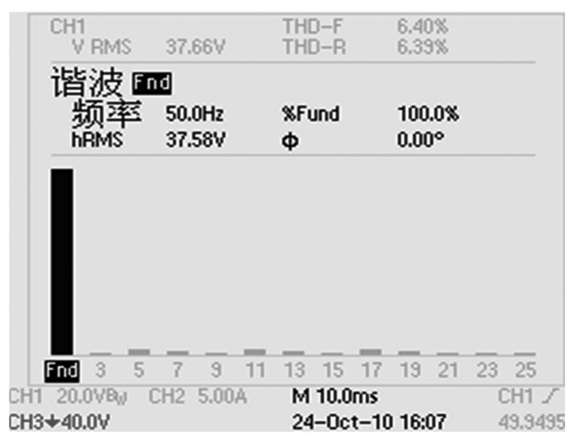


(c)

(d)



(e)



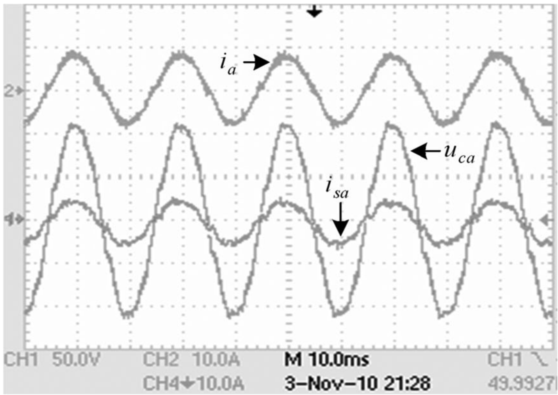
(f)

Fig. 14. Waveforms and FFT spectrum. (a) DC-link voltage, input voltage, and input current with reference output voltage of 33 V in Test 1. (b) FFT spectrum of input voltage with reference output voltage of 33 V in Test 1. (c) DC-link voltage, input voltage, and input current with reference output voltage of 42 V in Test 2. (d) FFT spectrum of input voltage with reference output voltage of 42 V in Test 2. (e) DC-link voltage, input voltage, and input current with reference output voltage of 42 V in Test 2. (f) FFT spectrum of input voltage with reference output voltage of 42 V in Test 3.

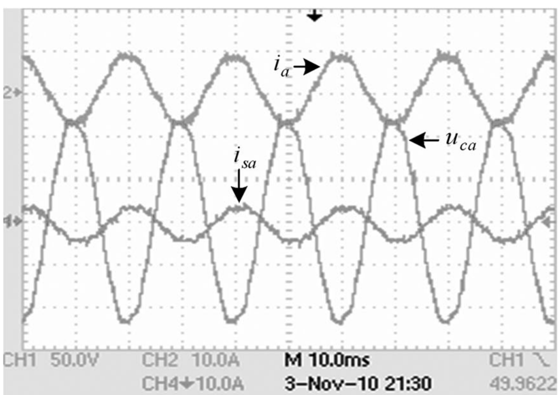
The second experiment, configured as shown in Fig. 7, is conducted to validate the effectiveness of the proposed methods to stabilize matrix converter in motor as well as generator mode in experiment. The parameters, used in this experiment, are the same as those in the corresponding simulation.

In this experiment, the constructive methods in (29) and (38) are used for the stabilization of matrix converter. When matrix converter operates in motor mode, the method in (29) is implemented; while the system runs in generator mode, the method in (38) is used. The mode is determined by the sign of reference current i_d. The parameters in (29) and (38) used in this experiment are $k = 1.0$, $\tau = 1.0$ ms, $k_\theta = -1.3$, $\tau_\theta = 1.0$ ms.

Without loss of generality, the reference current i_d is set to 8 A and the reference current i_q is assigned to be zero. The results



(a)



(b)

Fig. 15. Waveforms of input voltage, input current, and output current of matrix converter: (a) in motor mode and (b) in generator mode.

in steady state are shown in Fig. 15(a), where, u_a, i_{sa}, and i_a are the input voltage, input current, and the output current of matrix converter, respectively. Since i_q is set to zero and unity input power factor modulation is applied, all of them are in phase with each other.

When the reference current i_d is changed from 8 to -8 A, the energy is transferred from the inverter of matrix converter back to the rectifier; the results are shown in Fig. 15(b). It can be found that both the input current and the output current are in opposite phase with the grid voltage. It can also be observed that the input voltage of matrix converter in generator mode is slightly higher than that in motor mode. The experimental results show that the constructive methods do a good job for assuring the stability of matrix converter in motor as well as generator modes.

Based on the previous simulations and experimental results, it could be found that the proposed methods could improve the stability of matrix converter effectively, and behave better than the existed ones in stability and other performances. Especially, the results about stability in generator mode are important for wind energy conversion systems based on matrix converter.

IV. Conclusion

Inspired by the impedance ratio criterion, this paper proposes a constructive method by constructing a correction term to increase input impedance of matrix converter to improve the stability of the system. It is found that modifying the reference output voltage can change the input impedance of matrix converter on *D*-axis value, and modifying the rectifier modulation vector can change the input impedance on *Q*-axis. In motor mode, the input impedance on *D*-axis is negative; the matrix converter could be stabilized by just modifying the output reference properly, while in generator mode, the input impedance on *Q*-axis becomes negative, and the matrix converter could be stabilized by just modifying rectifier modulation vector according to the methods proposed in this paper. The small-signal stability of matrix converter under closed-loop modulation after using constructive method is proved. The constructive method is a generic method to improve stability. It could be applied to other power converter areas where there exists the similar oscillation problem.

Simulations and experiments are carried out, and they have verified the effectiveness of the proposed methods. In this paper, only the linear passive *RL* load and linear active load are investigated. Future work will concentrates on some complex nonlinear loads such as induction motor. The nonlinear constructive method should be studied in detail.

Appendix

The PI controller with feedforward for the active load in motor mode is designed as follows:

$$u_d^* = u_g - L\omega_s i_q + k_{pd}(i_d^* - i_d) + k_{id} z_d \tag{45}$$

$$\dot{z}_d = i_d^* - i_d \tag{46}$$

$$u_q^* = L\omega_s i_d + k_{pq}(i_q^* - i_q) + k_{iq} z_d \tag{47}$$

$$\dot{z}_q = i_q^* - i_q \tag{48}$$

where i_d^* are i_q^* are the desired active and reactive currents, respectively. u_g is the voltage on the left side of the transformer.

To stabilize the matrix converter, according to (29), the modified output voltages are

$$u_d^{**} = u_d^* + \frac{u_d^* f(u_{cd})}{\sqrt{(u_d^*)^2 + (u_q^*)^2}} \tag{49}$$

$$u_q^{**} = u_q^* + \frac{u_q^* f(u_{cd})}{\sqrt{(u_d^*)^2 + (u_q^*)^2}}. \tag{50}$$

Then the closed-loop dynamic equations of the active load could be described as

$$L\frac{di_d}{dt} = k_{pd}(i_d^* - i_d) + k_{id} z_d + \frac{u_d^* f(u_{cd})}{\sqrt{(u_d^*)^2 + (u_q^*)^2}} \tag{51}$$

$$L\frac{di_q}{dt} = k_{pq}(i_d^* - i_d) + k_{iq} z_d + \frac{u_q^* f(u_{cd})}{\sqrt{(u_d^*)^2 + (u_q^*)^2}}. \tag{52}$$

When the matrix converter works in generator mode, method (38) is used. The rectifier modulation vector angle could be directly solved by $\theta = \tan^{-1}(u_{c\beta}/u_{c\alpha})$, where $\theta \in [-\pi, \pi)$. However, it is not convenient to apply it to method (38) directly. So one simple observer is used, and it is written as

$$\dot{\hat{\theta}} = 100\pi + \frac{1}{\tau_\theta}(\theta - \hat{\theta}) \approx 100\pi + k_x \vec{u}_c \otimes e^{j\hat{\theta}} \tag{53}$$

where $k_x = 1/\tau_\theta \, |\vec{u}_c|$. From (53), the transfer function from θ to $\hat{\theta}$ could be described as

$$\frac{\hat{\theta}(s)}{\theta(s)} = \frac{1}{\tau_\theta s + 1} = 1 - \frac{\tau_\theta s}{\tau_\theta s + 1}\theta(s). \tag{54}$$

According to (54), we have

$$\theta(s) - \hat{\theta}(s) = \frac{\tau_\theta s}{\tau_\theta s + 1}\theta(s). \tag{55}$$

Then the correction term could be written as $g(\theta) = k_\theta(\theta - \hat{\theta})$. By the same way, the correction term for the output voltage could be described as $f(u_{cd}) = k(u_{cd} - \tilde{u}_{cd})$, where $\tilde{u}_{cd}(s) = (1/\tau s + 1)u_{cd}(s)$.

After linearization, the small-signal equations for the system are

$$\begin{aligned}
L_s \frac{d\Delta i_{sd}}{dt} &= -\Delta u_{cd} + L_s \omega_s \Delta i_{sq} - R_s \Delta i_{sd} \\
L_s \frac{d\Delta i_{sq}}{dt} &= -\Delta u_{cq} - L_s \omega_s \Delta i_{sd} - R_s \Delta i_{sq} \\
C_s \frac{d\Delta u_{cd}}{dt} &= \Delta i_{sd} + \omega_s \Delta u_{cq} - \chi \Delta u_{cd} + \frac{\bar{P}}{\bar{u}_{cd}^2}\alpha k \Delta \tilde{u}_{cd} \\
&\quad - \frac{1}{\bar{u}_{cd}}[(u_g - k_{pd}\bar{i}_d)\Delta i_d - k_{pq}\bar{i}_q \Delta i_q \\
&\quad + k_{id}\bar{i}_d \Delta z_d + k_{iq}\bar{i}_q \Delta z_q] \\
C_s \frac{d\Delta u_{cq}}{dt} &= \Delta i_{sq} - \omega_s \Delta u_{cd} \\
&\quad - \left[\frac{\bar{P}}{\bar{u}_{cd}^2}(1 + k_\theta)\Delta u_{cq} + \frac{\bar{P}}{\bar{u}_{cd}} k_\theta (\Delta\theta_s - \Delta\hat{\theta})\right] \\
\tau_\theta \Delta\dot{\hat{\theta}} &= \Delta\theta_s + \frac{\Delta u_{cq}}{\bar{u}_{cd}} - \Delta\hat{\theta} \\
\Delta\dot{\theta}_s &= \Delta\omega_s = 0 \\
\tau \Delta\dot{\tilde{u}}_{cd} &= \Delta u_{cd} - \Delta\tilde{u}_{cd} \\
L \frac{d\Delta i_d}{dt} &= -k_{pd}\Delta i_d + k_{id}\Delta z_d + \alpha k(\Delta u_{cd} - \Delta\tilde{u}_{cd}) \\
\Delta\dot{z}_d &= -\Delta i_d \\
L \frac{d\Delta i_q}{dt} &= -k_{pq}\Delta i_q + k_{iq}\Delta z_q + \beta k(\Delta u_{cd} - \Delta\tilde{u}_{cd}) \\
\Delta\dot{z}_q &= -\Delta i_q
\end{aligned}$$

where $\chi = (\bar{P}/\bar{u}_{cd}^2)(\alpha k - 1), \alpha = \bar{u}_d^* f(\bar{u}_{cd})/\sqrt{(\bar{u}_d^*)^2 + (\bar{u}_q^*)^2}$, $\beta = \bar{u}_q^* f(\bar{u}_{cd})/\sqrt{(\bar{u}_d^*)^2 + (\bar{u}_q^*)^2}, \bar{P} = \bar{u}_d^* \bar{i}_d$. According to these equations, the stability analysis, which is the same as that in the previous section, could be performed. If only the method (29) is used, k_θ in the aforementioned equations should be set to zero; if only method (38) is used, k should be set to zero.

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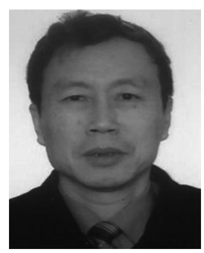

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